

Theorem 1.1

A function $y = f(x)$ satisfies the equation

$$\frac{dy}{dx} = ky \text{ or } f'(x) = k \cdot f(x)$$

if and only if

$$y = ce^{kx} \text{ or } f(x) = ce^{kx}$$

Find the function that satisfies the following equations:

$$1 \quad \frac{dA}{dt} = 5A$$

$$\frac{dP}{dt} = 12P$$

Definition 1.2

Uninhibited Population Growth

$$P(t) = P_0 e^{ky} \text{ where } t \text{ is time.}$$

Note, when $t = 0$,

$$P(0) =$$

k is called the

Example 1.3

An investment of P_0 is invested with the interest compounded continuously at 9.15% per year.

- 1 Find a function that satisfies $\frac{dP}{dt} = 0.0915P$.
- 2 Suppose \$20000 is invested. What is the balance after 3 years?
- 3 How long will it take for this investment to double?

Theorem 1.4

The exponential **growth rate** k and the **doubling time** T are related by

$$kT = \ln 2$$

which means

Membership in Facebook has been doubling every 6 months. What is the exponential growth rate of Facebook membership, as a percentage?

Example 1.5

A 1939 comic book with the first appearance of Batman sold at auction in 2010 for \$1,075,000. The comic originally cost \$0.10. Suppose the value V of the comic book grew exponentially, as given by

$$\frac{dV}{dt} = kV$$

- 1 Find a function that satisfies this equation. Let $V_0 = \$0.10$.
- 2 Estimate the value in 2020.
- 3 What is the doubling time for the value of the comic book?
- 4 In what year will the value of the comic book be \$30 million?

Models of Limited Growth

Definition 1.6

$$P(t) = \frac{L}{1 + be^{-kt}}$$

, where L is the limit on growth and $k > 0$.

A graph looks like this:

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Example 1.7

On a college campus of 5000 students, one student returned from vacation with a contagious flu virus. The spread of the virus through the student body is given by

$$P(t) = \frac{5000}{1 + 4999e^{-0.8t}}$$

where $P(t)$ is the total number of infected students after t days. The college will cancel classes when 40% or more of the students are ill.

- 1 How many students will be infected after 1 day? 3 days? 5 days?

- 2 After how many days will the college cancel classes?
- 3 What is the reate of infection on this day?