

## Definition 1.1

A **logarithm** is defined as follows:

## Theorem 1.2 (Properties of Logarithms)

For any positive numbers  $M$ ,  $N$ ,  $a$ , and  $b$ , with  $a, b \neq 1$ , and  $k$  is any real number.

$$1 \log_a(MN) =$$

$$2 \log_a\left(\frac{M}{N}\right) =$$

$$3 \log_a M^k =$$

$$4 \log_a a =$$

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## Theorem 1.3 (Properties of Logarithms, continued)

$$5 \log_a(a^k) =$$

$$6 \log_a 1 =$$

$$7 \log_b M =$$

*Change of Base Formula*

## Example 1.4

Given  $\log_a 2 = 0.301$ ,  $\log_a 5 = 0.699$ , and  $\log_a 6 = 0.778$ , find each of the following:

$$1 \log_a 3$$

$$2 \log_a \frac{1}{3}$$

3  $\log_a 50$

4  $\log_a 6a^3$

5  $\log_a \sqrt[3]{5}$

6  $\log_2 5$

### Definition 1.5

Common Logarithm:

Natural Logarithm:

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## Graph of $\ln x$

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**Example 1.6**

Solve the following:

**1**  $\log_2 16 =$

**2**  $\log_9 3 =$

**3**  $e^t = 150$

**4**  $e^{0.08t} = 2500$

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**Theorem 1.7 (Derivative of the Natural Logarithmic Function)**

$$\frac{d}{dx} \ln x =$$

**Example 1.8**

Find the derivative of the following functions:

**1**  $f(x) = x^2 \ln x$

$g(x) = 6 (\ln x)^3$

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$$2 \quad y = \frac{\ln x}{x^2}$$

Theorem 1.9 (Derivative of the Natural Logarithmic Function, part 2)

$$\frac{d}{dx} \ln(f(x)) =$$

$$1 \quad \text{Find } \frac{d}{dx} \ln(13x) - \frac{1}{3}x^3$$

$$\frac{d}{dx} \ln(x^2 + 1)$$

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$$2 \quad \text{Find } \frac{d}{dx} (\ln x)^{15}$$

$$\frac{d}{dx} \ln(\ln 2x)$$

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3 Find  $\frac{d}{dx} \ln \sqrt{1+x^3}$

$$\frac{d}{dx} \ln \left( \frac{x^2 - 7}{x} \right)$$

4  $\frac{d}{dx} \log_3(2x + 1)$

$$\frac{d}{dx} [(x^2 + 1)(x - 3)]$$

**Example 1.10**

Students in a math class took a final exam. They took equivalent forms of the exam at monthly intervals thereafter. After  $t$  months, the average score  $S(t)$ , as a percentage, was found to be given by

$$S(t) = 78 - \ln(t - 1)$$

- 1 What was the average score when they initially took the test,  $t = 0$ .
- 2 What was the average score after 4 months?
- 3 What was the average score after 24 months?
- 4 Find  $S'(t)$
- 5 Find the maximum and minimum values, if they exist.
- 6 Find  $\lim_{t \rightarrow \infty} S'(t)$