Definition 1.1

Let $C(x)$, $R(x)$, and $P(x)$ represent, respectively, the total cost, revenue, and profit from the production and sale of x items.

The **marginal cost** at x, given by $C'(x)$, is the approximate cost of the $(x+1)$st item:

$$C'(x) \approx C(x+1)C(x) \text{ or } C(x+1) \approx C(x) + C'(x)$$

The **marginal revenue** at x, given by $R'(x)$, is the approximate revenue from the $(x+1)$st item:

$$R'(x) \approx R(x+1)R(x) \text{ or } R(x+1) \approx R(x) + R'(x)$$

The **marginal profit** at x, given by $P'(x)$, is the approximate profit of the $(x+1)$st item:

$$P'(x) \approx P(x+1)P(x) \text{ or } P(x+1) \approx P(x) + P'(x)$$

Example 1.2

A company estimates that the cost in dollars of producing x units of crazy straws is given by $C(x) = 800 + 0.04x + 0.0002x^2$. Find the production level that minimizes the average cost per straw.

Example 1.3

Cost of producing x deep-tread radial tires is $C(x) = 4000 + 70x - 0.01x^2$ dollars and revenue from the sale of x deep-tread radial tires is $R(x) = 105x - 0.02x^2$ dollars.

1. Determine the marginal cost.
2. Determine the marginal revenue.
3. Determine $R'(50)$
4. What does $R'(50)$ mean?
5. Determine the profit function.
6. Determine marginal profit.
7. For what value of x is the marginal cost equal to the marginal revenue, and what is the marginal profit in this instance?

Differentials

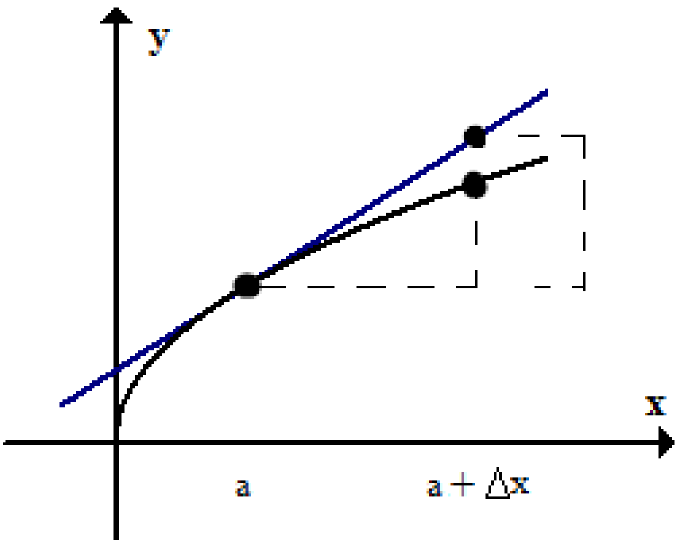
Differentials help us estimate the change in function values. Let's look at some new notation.

1. Δx - is the true change in x
2. differential dx - is our independent variable that represents the chagen in x. We let $dx = \delta x$.
3. Δy - is the true change in y
4. differential dy - is the estimated change in y

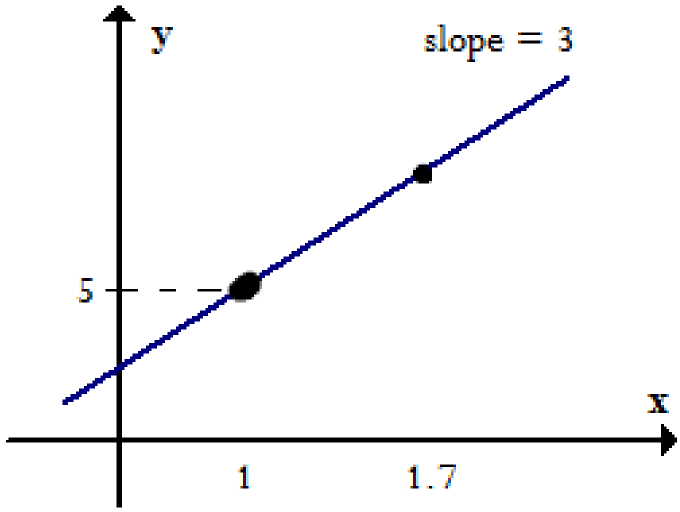
$$\frac{dy}{dx} = f'(x)$$

This was another way of notating the derivative. Now if we treat dx as an independent variable, we can rearrange this as

$$dy = f'(x) \cdot dx$$



Let's look at a simple example. Suppose I know at $x = 1$, the y-value is 5. How can I use the differential formula to estimate $f(1.7)$?



Back to the Tires

5. Find the actual change in revenue as you go from 50 tires to 51 tires, and compare this result with $R'(50)$.

Example 1.4

Let $y = f(x) = x + x^2$. Find dy and Δy when $x = 3$ and $\Delta x = 0.2$. Do it again for $\Delta x = -.2$