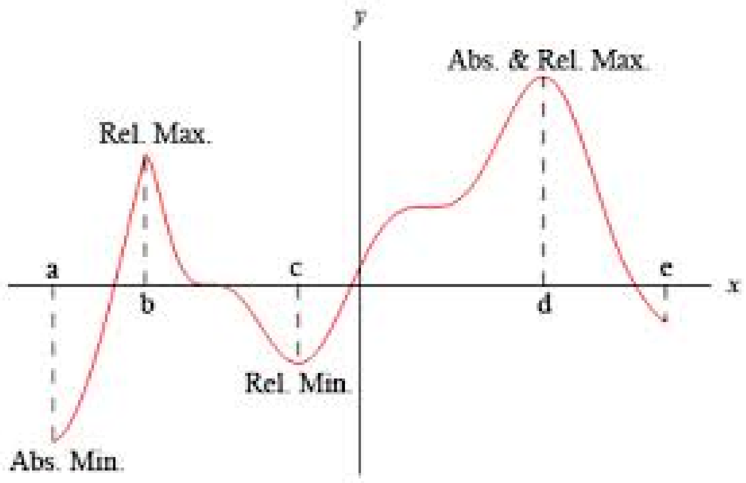
What is an Absolute Maximum / Minimum



Theorem 1.1 (The Extreme Value Theorem)

A continuous function f on a closed interval $[a, b]$ must have an absolute maximum and an absolute minimum value over $[a, b]$.

Maximum-Minimum Principle 1

Suppose that f is continuous over $[a, b]$. To find the absolute maximum and minimum values over $[a, b]$:

1. Find $f'(x)$
2. Find all critical values in $[a, b]$. If a critical value is outside $[a, b]$, discard it.
3. List the critival values from (2) and the endpoints a and b:

$$a, c_1, c_2, ..., c_n, b$$

4. Evaluate $f(x)$ for each value from (3):

$$f(a), f(c_1), f(c_2), ..., f(c_n), f(b)$$

The largest of these is the absolute maximum. The smallest of these values is the absolute minimum.

Example 1.2

Find the absolute extrema of

1. $f(x) = -x^3 + 3x^2 - 2$ over $[-3, 3]$
2. $f(x) = -x^3 + 3x^2 - 2$ over $[-2, 1]$
3. $f(x) = 4x - x^2$
4. $f(x) = 4x - x^2$ over $[1, 4]$

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Maximum-Minimum Principle 2

Suppose that f is a function such that $f'(x)$ exists on an interval I and there is **EXACTLY ONE** critical value in c where $f'(c) = 0$.

$$\text{If } f''(c) < 0, \text{ then } f(c) \text{ is the absolute maximum over } I$$

or

$$\text{If } f''(c) > 0, \text{ then } f(c) \text{ is the absolute minimum over } I$$

Example 1.3

Find the absolute extreme of $f(x) = x + \dfrac{1}{x}$ over $(0, \infty)$.

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Example 1.4

Find the absolute maximum and minimum values of $f(x) = (x-2)^3$ over $(-\infty, \infty)$

Find the absolute maximum and minimum values of $f(x) = (x-2)^3$ over $[0, \infty)$