Business Calculus

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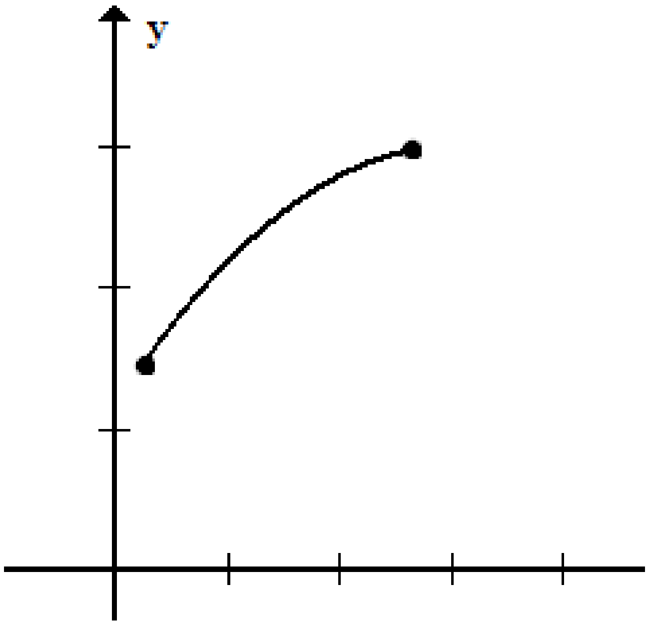
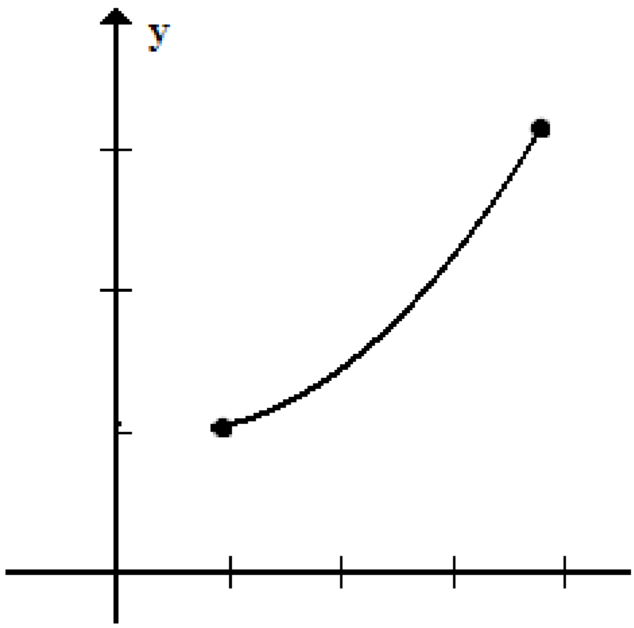
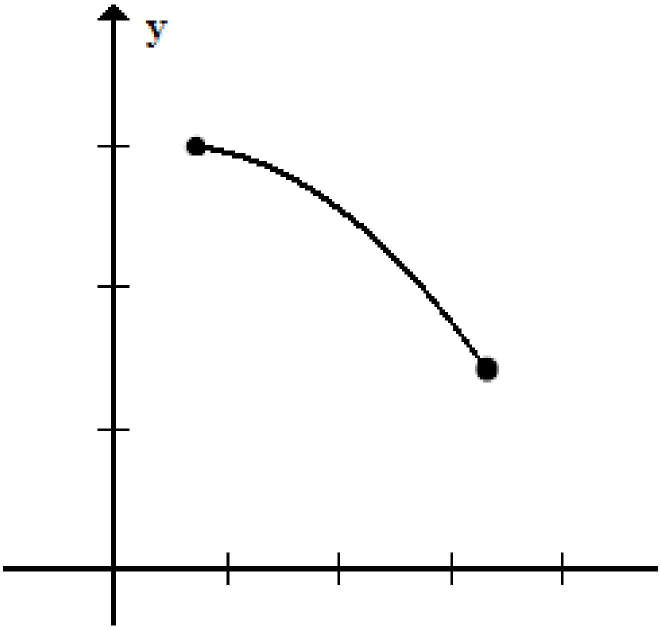
Suppose we have two increasing functions.

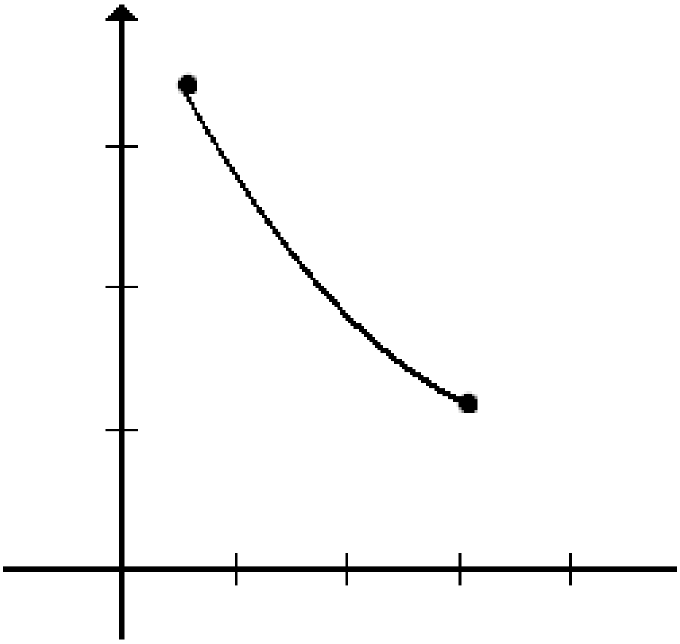




How can I determine the way the function curves?

Suppose we have two decreasing functions.



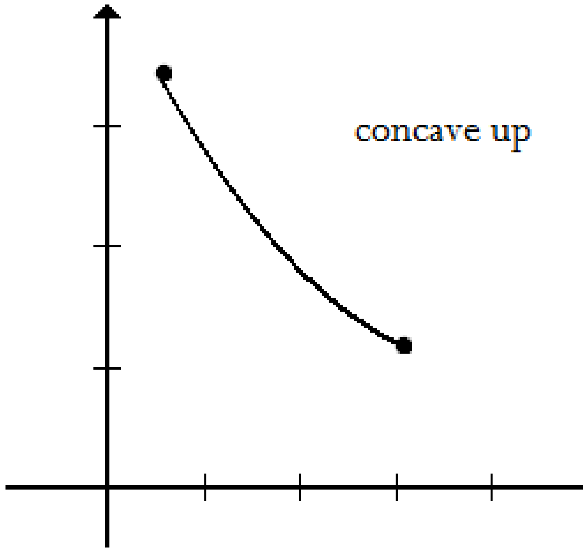


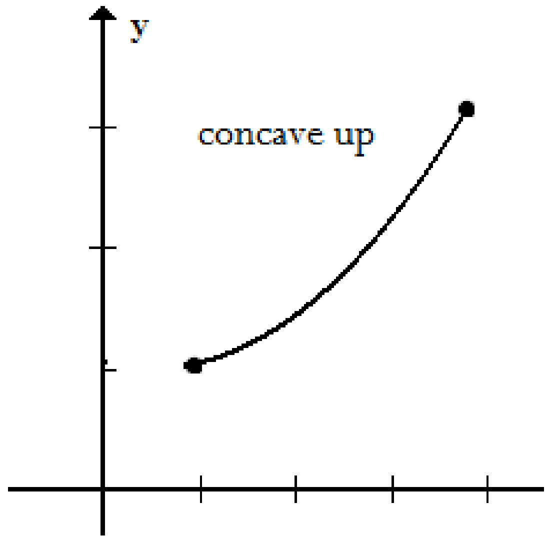
How can I determine the way the function curves?

A Test for Concavity

Theorem 1.1

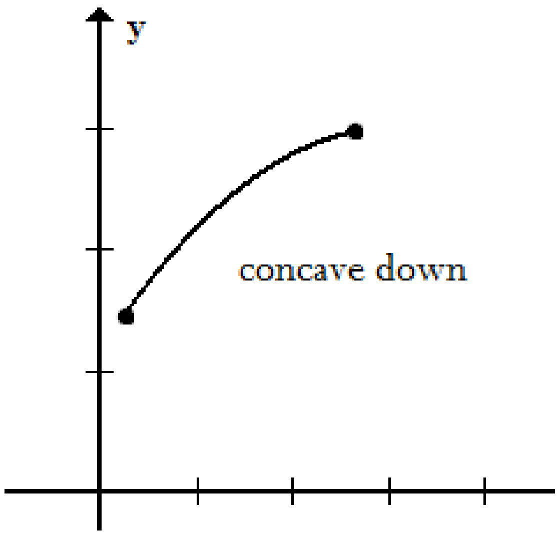
1. *If $f''(x) > 0$ on an interval I, then the graph of f is* ***CONCAVE UP****.*

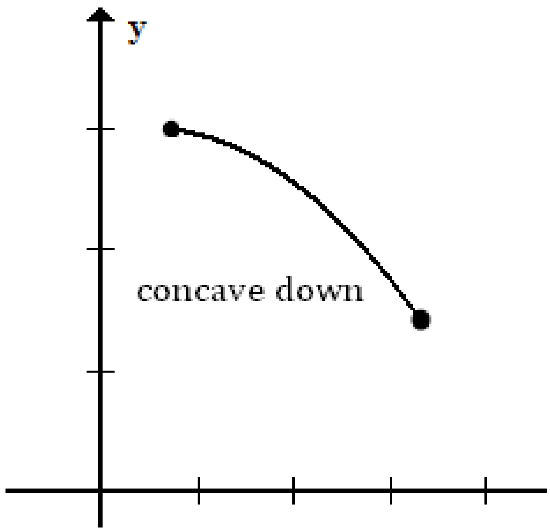




Theorem 1.2

2. *If $f''(x) < 0$ on an interval I, then the graph of f is* ***CONCAVE DOWN****.*





The Second Derivative Test for Relative Extrema

Theorem 1.3

Suppose f is differentiable on (a, b) and there is a critical value at $x = c$ where $f'(c) = 0$. Then

1. *If $f''(c) > 0$, then $f(c)$ is a relative minimum.*

2. *If $f''(c) < 0$, then $f(c)$ is a relative maximum.*

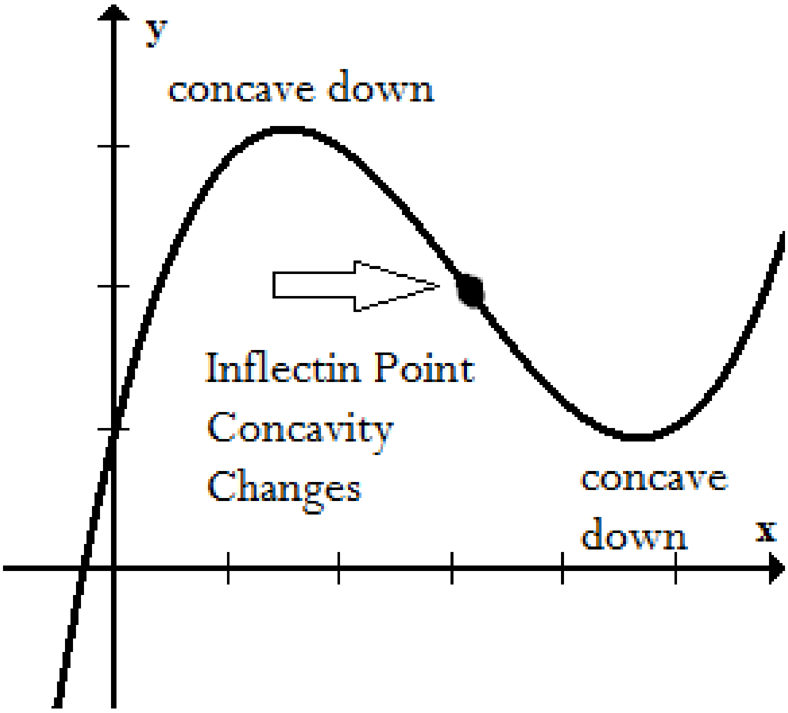
Example 1.4

Determine where the function $f(x) = 2x^3 - 3x^2 - 12x$ is concave up or concave down.

Inflection Point

Definition 1.5

A point on $f(x)$ is called an **Inflection Point** if $f(x)$ is continuous and it changes concavity.



Theorem 1.6 (Finding Points of Inflection)

If a function f has a point of inflection, it must occur at a point $x = c$, where

$$f''(c) = 0 \text{ or } f''(c) \text{ does not exist.}$$

Go back to $f(x) = 2x^3 - 3x^2 - 12x$.

Summary of Curve Sketching

1. Domain of $f(x)$
2. x and y intercepts
   1. x-intercepts occur when $f(x) = 0$
   2. y-intercept occurs when $x = 0$
3. Find $f'(x)$
   1. Find the critical values, all x-values where $f'(x) = 0$ or when $f'(x)$ does not exist. **These are potential local extrema.**
   2. Find increasing / decreasing intervals using numberline
   3. Find local maximums / minimums (if any exist). Remember to write them as points.
      1. Local Max at $x = c$: $f'(x)$ changes from (+) to (-) at $x = c$.
      2. Local Min at $x = c$: $f'(x)$ changes from (-) to (+) at $x = c$.
      3. You can also use the **Second Derivative Test** to determine local extrema by plugging $x = c$ into $f''(x)$.

4. Plot them
5. Find $f''(x)$
   1. Find all x-values where $f''(x) = 0$ or when $f''(x)$ does not exist. **These are potential inflection points.**
   2. Find intervals of concavity using the number line
      1. If $f''(x) > 0$, f is concave up.
      2. If $f''(x) < 0$, f is concave down.
   3. Find points of inflection
      1. Must be a place where concavity changes
      2. The point must exist (i.e, can't be an asymptote, discontinuity)
   4. Plot them
6. Sketch

Example 1.7

Sketch $f(x) = 2x^3 - 3x^2 - 12x$ by finding local extrema, intervals of increasing / decreasing, concavity, and inflection points.

Example 1.8

Sketch $f(x) = 5x^3 - 3x^5$ by finding local extrema, intervals of increasing / decreasing, concavity, and inflection points.

Example 1.9

Sketch $f(x) = x^{1/3}(x + 4)$ by finding local extrema, intervals of increasing / decreasing, concavity, and inflection points.

Chapter 2 - Applications of Differentiation 2.2 - Using Second Derivatives to Find Local Extrema

Example 1.10

Graph $f(x) = x^3 - 3x^2 + 1$, $f'(x)$, and $f''(x)$ on the same graph. Any connections?