Business Calculus

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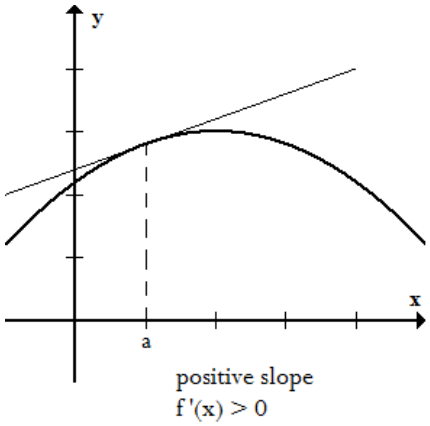
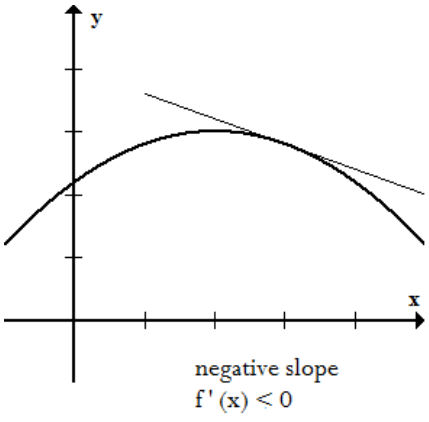
Increasing and Decreasing Functions

Consider the following two graphs

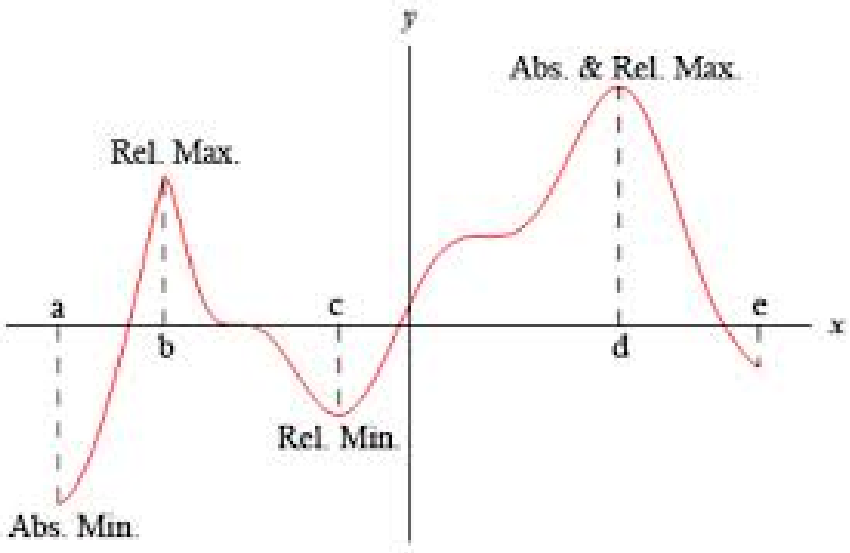




Increasing:

Decreasing:

Critical Values



Definition 1.1

A **critical value** is any number $x = c$ where

Theorem 1.2

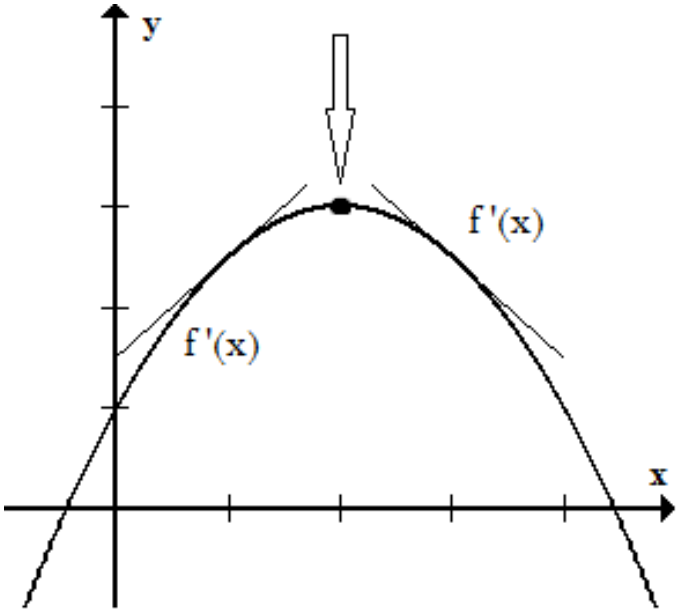
If a function f has a local minimum or local maximum at $x = c$, then c is a critical value

Example 1.3

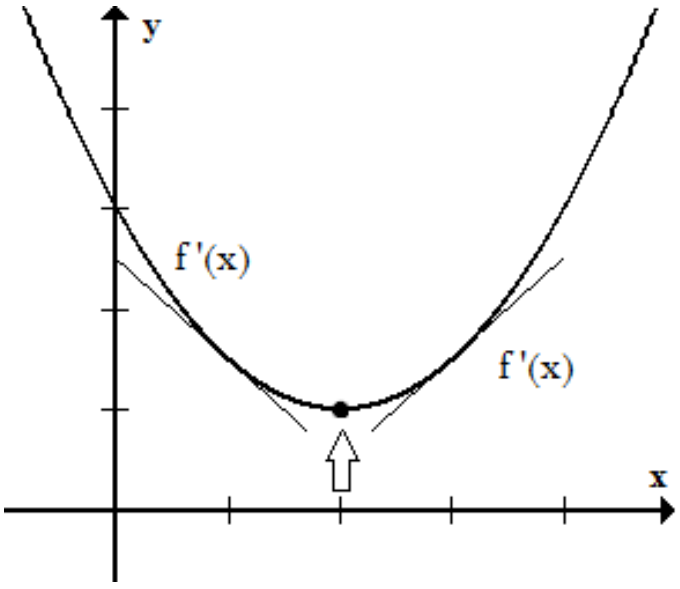
Find the critical points of $f(x) = \frac{1}{3}x^3 + 3x^2 + 9x + 2$

First Derivative Test for Relative Extrema

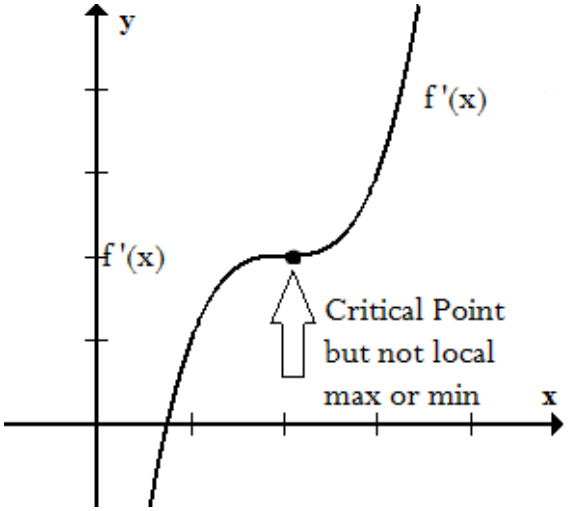
1. If $f'(x)$ changes from a postive $(+)$ to a negative $(-)$ at $x = c$, then $f(x)$ has a local maximum at $x = c$.

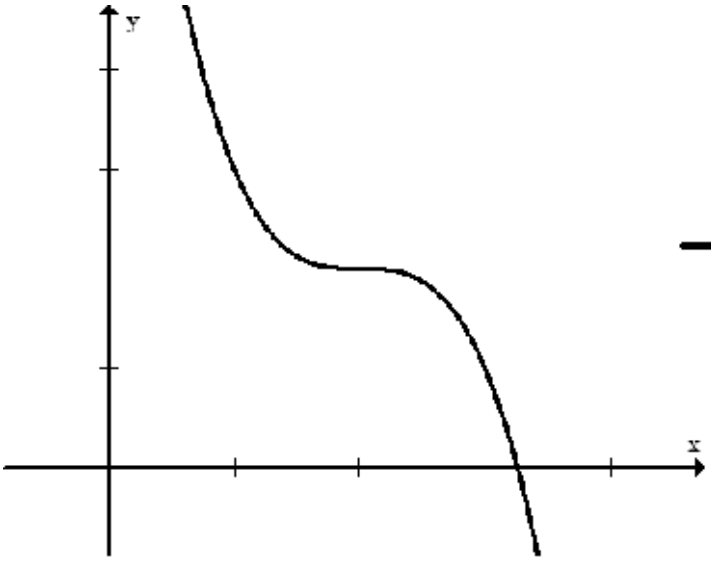


2. If $f'(x)$ changes from a negative $(-)$ to a positive $(+)$ at $x = c$, then $f(x)$ has a local minimum at $x = c$



3 If $f'(x)$ does not change signs, $f(x)$ has neither a local maximum nor local minimum at $x = c$.





Example 1.4

Let $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$. Find where $f(x)$ is increasing / decreasing, and identify any local extrema. Then sketch the graph.

Example 1.5

Let $f(x) = x^4 - 4x^3$. Find where $f(x)$ is increasing / decreasing, and identify any local extrema. Then sketch the graph.

Example 1.6

Let $f(x) = x^{1/3}(x+4)$, $f'(x) = \dfrac{4(x+1)}{3x^{2/3}}$. Find where $f(x)$ is increasing / decreasing, and identify any local extrema. Then sketch the graph.

Example 1.7

Suppose that $P(x) = -0.01x^2 + 60x - 500$ is the profit function from the manufacture and sale of crazy straws. Is profit increasing or decreasing when 100 crazy straws are sold? What about 5000 crazy straws? What is the optimal number of crazy straws to sell?