Business Calculus

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Why we need the chain rule...

Example 1.1

Consider the difference between the following functions

1. $y = 2x^3$ and $y = 2(5x + 7)^3$
2. $y = \sqrt{x}$ and $y = \sqrt{x^2 + 1}$
3. $y = x^5$ and $y = \left(\dfrac{x-1}{x+1}\right)^5$

Theorem 1.2 (The Extended Power Rule)

Suppose $g(x)$ is a differentiable function. Then, for any real number n

$$\frac{d}{dx}\left[g(x)\right]^n =$$

1. $y = 2(5x+7)^3$

2. $y = \sqrt{x^2+1}$

3. $y = \left(\frac{x-1}{x+1}\right)^5$

4 $y = 6x(x^2 + x)^4$

5 $y = (3x - 7)^{12} \cdot (x - 1)^4$

6 $y = \dfrac{(8x - 1)^6}{5x + 4}$

Theorem 1.3 (The Chain Rule)

Let f and g be differentiable functions. Then

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] =$$

Theorem 1.4 (Other Version of Chain Rule)

Let $y = f(u)$ and $u = g(x)$. Then

$$\frac{d}{dx}\left[f(g(x))\right] =$$

Example 1.5

Find an equation for the tangent line to the graph of $y = \sqrt{x^2 + 3x}$ at the point (1,2).

Multiple Chain Rule

Example 1.6

Differentiate $y = \left(3x^2 + 3(6x^2 - 1)^3\right)^5$