

Business Calculus

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Chapter 1 - Differentiation 1.6 - The Product and Quotient Rule

Theorem 1.1 (The Product Rule)

Let $F(x) = f(x) \cdot g(x)$. Then the derivative

$$F'(x) = \frac{d}{dx} [f(x) \cdot g(x)] =$$

Example 1.2

Differentiate the following functions.

1 $F(x) = (x^3 + 8x)(5 - x^4)$

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$$2 \quad f(x) = (\sqrt{x} + 1) \left(18x - \frac{1}{x}\right)$$

Theorem 1.3 (The Quotient Rule)

If $F(x) = \frac{f(x)}{g(x)}$, then the derivative is

$$F'(x) = \underline{\hspace{15em}}$$

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Example 1.4

Find the derivative of $f(x) = \frac{x^3 - 2x^2 + 5}{3 - 7x}$

Example 1.5

Find the equation of the line tangent to the graph of $f(x) = \frac{x}{x-2}$ when $x = 3$.

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Example 1.6

Find $\frac{d}{dx} \left[\frac{4x}{5+2x} - 5x^2 \right]$

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Example 1.7

Find the derivative to $f(x) = \frac{3x^2 + 4x + 3}{\sqrt{x}}$ (two different ways)

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Example 1.8

Find the derivative to $y = 3x^2 \left(5x - \frac{3}{x^4} + \frac{1}{3x^2} \right)$ (two different ways)

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Example 1.9

The population P , in thousands, of a small city is given by $P(t) = \frac{500t}{2t^2 + 9}$, where t is time in months.

- 1 Find the growth rate.
- 2 Find the population after 12 months.
- 3 Find the growth rate after 12 months.
- 4 Estimate the population at 13 months.

Definition 1.10

1 $C(x)$

2 $\frac{C(x)}{x}$

3 $R(x)$

4 $\frac{R(x)}{x}$

5 $P(x)$

6 $\frac{P(x)}{x}$

Example 1.11

Suppose you sell 12 items. It costs \$24 to produce the items. Your revenue is \$60. What is your profit, average profit, average cost, and average revenue?

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Example 1.12

The cost, in dollars, of producing x bikes is given by $C(x) = 200 + 100x^{1/4}$. If revenue is $R(x) = 120 + 90\sqrt{x}$

- 1 Find the average cost, average revenue, and average profit.
- 2 Find the rate at which the average profit is changing when 20 bikes are being sold.

