

Business Calculus

Brian E. Veitch

Northern Illinois University

January 19, 2014

1 / 13

Chapter 1 - Differentiation 1.5 - Power Rule and Sum - Difference Rules

Notations of the Derivative

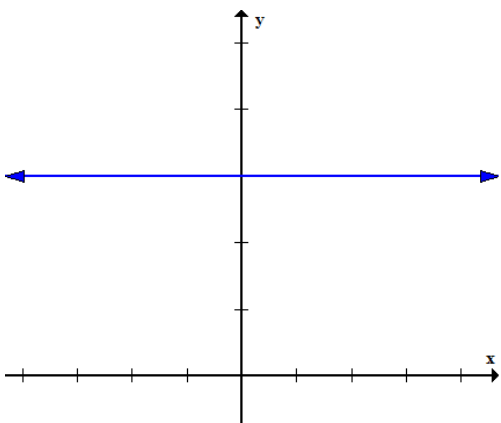
The derivative of $y = f(x)$ may be written in any of the following ways:

$$f'(x), \quad y', \quad \frac{dy}{dx}, \quad \frac{d}{dx} [f(x)]$$

Theorem 1.1 (Constant Rule)

If $f(x) = c$, where c is any real number, then

The derivative of a constant is 0.



2 / 13

Power Rule

Theorem 1.2 (Power Rule)

If $f(x) = x^n$ for any real number n , then

$$f'(x) =$$
$$\frac{d}{dx}x^n =$$

Bring down the exponent and subtract 1 from the exponent.

Example 1.3

Differentiate the following functions

- 1 $y = x^7$
- 2 $y = 5x^7$
- 3 $f(x) = x$
- 4 $f(x) = \frac{4}{7}x$
- 5 $f(x) = x^{16/7}$

Writing $f(x)$ in the Correct Form

In order to use the **Power Rule**, $f(x)$ needs to be in the form

$$f(x) = cx^n$$

where its derivative is

$$f'(x) = c \cdot nx^{n-1}$$

Example 1.4

Find the derivative of the following functions.

■ $f(x) = \sqrt{x}$

■ $f(x) = 4x^{3/7}$

5 / 13

■ $f(x) = \frac{1}{2\sqrt[3]{x}}$

■ $f(x) = \frac{5}{x^8}$

6 / 13

Theorem 1.5 (The Sum-Difference Rule)

Sum - The derivative of a sum is the sum of the derivatives:

$$\frac{d}{dx} [f(x) + g(x)] =$$

Difference - The derivative of a difference is the difference of the derivatives:

$$\frac{d}{dx} [f(x) - g(x)] =$$

Example 1.6

Differentiate the following functions. (Find $\frac{dy}{dx}$)

■ $y = -x^2 + .5x - 8$

$$y = 2x^{-3/4} - \frac{2}{x^2} + \sqrt[3]{x^4}$$

7 / 13

Example 1.7

Given $y = x + \frac{2}{x^3}$, find $\left. \frac{dy}{dx} \right|_{x=1}$

8 / 13

Example 1.8

Find an equation of the tangent line to the graph of $y = x + \frac{2}{x^3}$ at $(1, 3)$.

Growth of a Baby

Example 1.9

The median weight of a boy whose age is between 0 and 36 months can be approximated by the function

$$w(t) = 8.15 + 1.82t - 0.0596t^2 + 0.000758t^3$$

where t is measured in months and w is measured in pounds.

- 1 Find the rate of change of weight with respect to time.
- 2 Find the weight of the baby at age 10 months.
- 3 Find the rate of change of the baby's weight with respect to time at 10 months.
- 4 Find the rate of change at 11 months, 15 months, 20 months.

Example 1.10

Find the points on the graph at which the tangent line is horizontal.

1 $y = x^3 - 2$

2 $y = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x$

3 $y = -4x + 5$