Business Calculus

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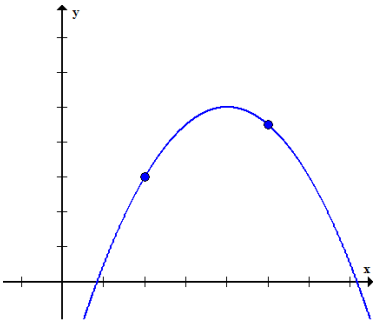
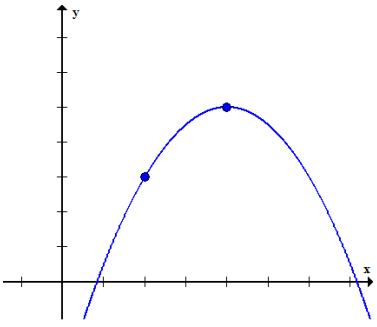
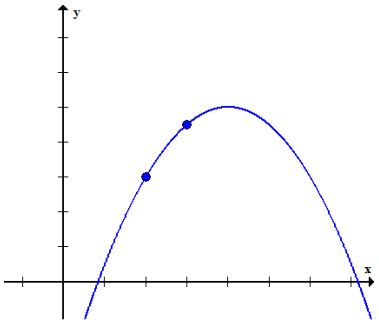
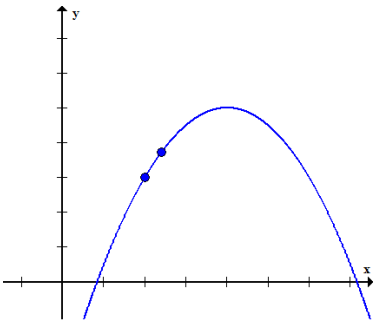
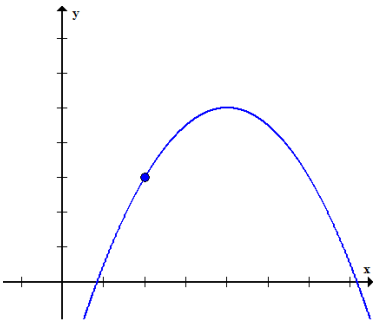
Secant Lines and the Tangent Line



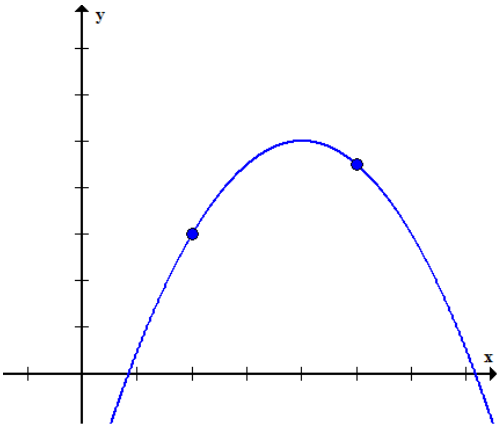


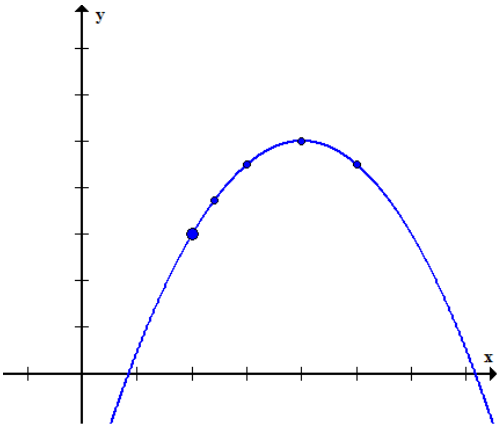






Definition of the Tangent Line and Instantaneous Rate of Change





$$m_{secant} = \frac{f(x+h) - f(x)}{h}$$

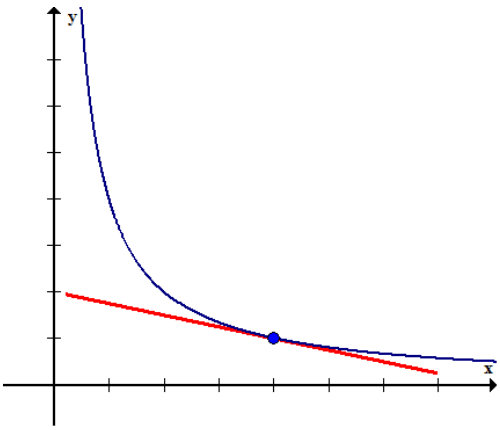
Definition 1.1

The slope of the tangent line is

This is also called the **instantaneous rate of change** of $f(x)$ at x.

Example 1.2

Let $f(x) = \frac{4}{x}$. We found the slope of several secant lines in the last section. Find the slope of the tangent line at $x = 2$.



Definition 1.3

For a function $y = f(x)$, its **derivative** at x is the function f' defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists. If $f'(x)$ exists, we say f is **differentiable**.

The steps to calculate the derivative.

1. Write down the **difference quotient**. $\dfrac{f(x+h) - f(x)}{h}$
2. Simplify the difference quotient.
3. Evaluate the limit as $h \to 0$. In other words, plug in $h = 0$, assuming you can.

Example 1.4

Graph $f(x) = -4x + 3$. Draw the tangent lines at $x = -1, 1, 3$. Find $f'(x)$ and then find $f'(-1)$, $f'(1)$, and $f'(3)$.

Example 1.5

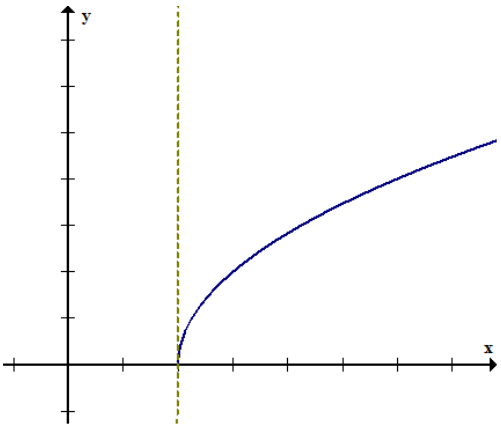
Graph $f(x) = x^2 - 3x$. Draw the tangent lines at $x = -1, 1, 3$. Find $f'(x)$ and then find $f'(-1)$, $f'(1)$, and $f'(3)$.

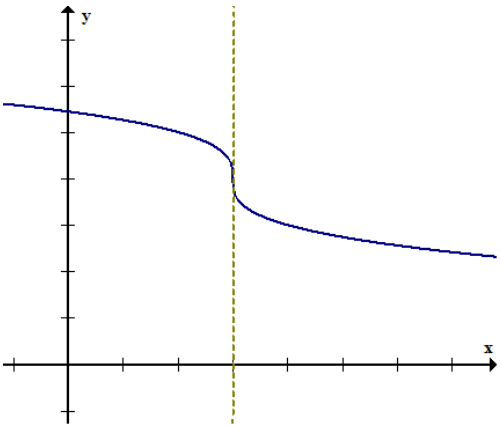
Example 1.6

Let $f(x) = 2\sqrt{x}$. Find $f'(x)$. Then find an equation of the tangent line to the graph at $(1/4, 1)$, $(1, 1)$, and $(9, 6)$.

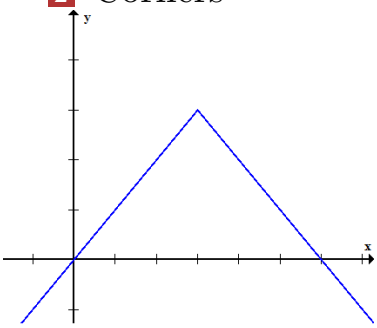
When does a function fail to be differentiable?

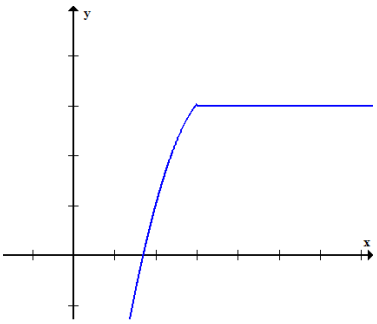
1 Vertical Tangent Lines





2 Corners





3 Discontinuities (asymptotes, jumps, holes)