

# Business Calculus

Brian E. Veitch

Northern Illinois University

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Chapter 1 - Limits 1.3 - Average Rates of Change

## Example 1.1

If I bicycle 16.4 miles in 45 minutes, what is my average speed?

## Example 1.2

From 1986 to 1991, Amtrak's annual revenue increased from \$861,000,000 to \$1,359,000,000. What is the average rate of change in revenue per year during this time?

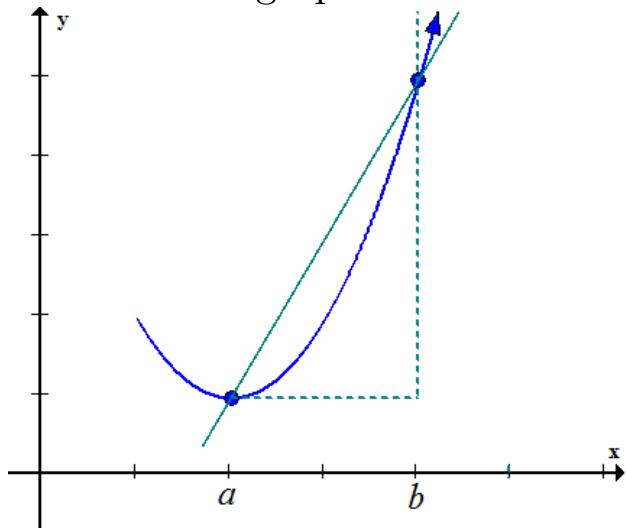
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# Average Rate of Change

## Definition 1.3

The **average rate of change** of  $f(x)$  with respect to  $x$  for a function  $f$  as  $x$  changes from  $a$  to  $b$  is

Consider the graph



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## Example 1.4

Find the average rate of change for  $f(x) = 2x^2 - 6x + 4$  on the indicated intervals.

1  $[0, 6]$

2  $[4, 6]$

3  $[5, 6]$

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### Example 1.5

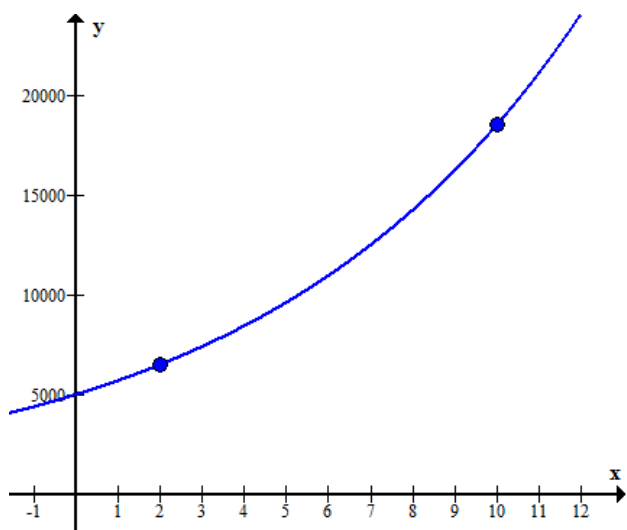
When a balance of \$5000 is owed on a credit card and interest is being charged at a rate of 15% per year, the total amount owed after  $t$  years,  $A(t)$ , is given by

$$A(t) = 5000(1.15)^t$$

- 1 Find  $A(4)$
- 2 Find  $A(2)$
- 3 Find  $A(4) - A(2)$
- 4 Find  $\frac{A(4) - A(2)}{4 - 2}$
- 5 What does the answer to the last question mean?

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### Regarding the last question



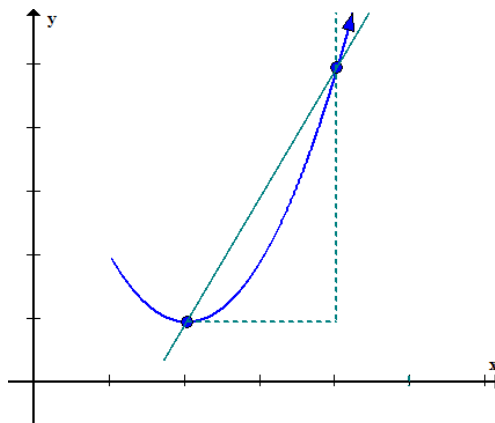
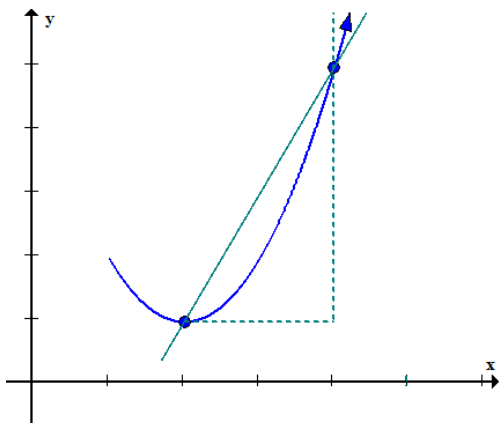
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# The Difference Quotient

## Definition 1.6

The average rate of change of  $f$  with respect to  $x$  is also called the **Difference Quotient** given by

$$\frac{f(x+h) - f(x)}{h}, \text{ where } h \neq 0$$



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## Example 1.7

Let  $f(x) = x^2 - 5x$ . Find the difference quotient when

- 1  $x = 3$  and  $h = 2$
- 2  $x = 3$  and  $h = 1$
- 3  $x = 3$  and  $h = 0.5$

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**Example 1.8**

Find the simplified form of the difference quotient. Then answer the previous questions again.

## Example 1.9

Find the simplified form of the difference quotient for  $f(x) = \frac{4}{x}$  and complete the following table.

$x$	$h$	$\frac{f(x+h) - f(x)}{h}$
4	2	
4	1	
4	.1	
4	.01	

**Example 1.10**

Find the simplified form of the difference quotient for  $f(x) = \sqrt{3x - 1}$ .