Business Calculus

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Theorem 1.1

If $f(x)$ is a rational function (polynomials are rational) or any "nice" function and a is in the domain of f, then

$$\lim_{x \to a} f(x) = f(a)$$

Example 1.2

$\lim_{x \to 3} \frac{x}{x+2}$

Example 1.3

$\lim_{x \to 4} \sqrt{16 - x^2}$

Limit Principles

Suppose $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$

1. $\lim_{x \to a} f \pm g = L \pm M$
   The limit of a sum or difference of functions is the sum or difference of their limit values.

2. $\lim_{x \to a} c \cdot f(x) = c \cdot L = c \cdot \lim_{x \to a} f(x)$

   The constant can be pulled out of a limit.

3. $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L \cdot M$

   Please note that you can only break a limit of a product into a product of limits provided the individual limits both exist.

4. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{L}{M}$, provided $\lim_{x \to a} g(x) \neq 0$.

Limit Principles

1. $\lim_{x \to a} c = c$

   This should make sense. We are taking a limit of a constant. No matter what x is, the function value is always c.

2. There are a few more in the textbook.

Example 1.4

Find $\lim_{x \to 5} 17$ and $\lim_{x \to -\infty} 17$

Evaluating Limits

Example 1.5

Find $\lim_{x \to -3} \dfrac{2x^2 + 5x - 3}{x^2 - 9}$

Example 1.6

Consider the limit $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$

Continuity

Definition 1.7

A function is continuous at $x = a$ if

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

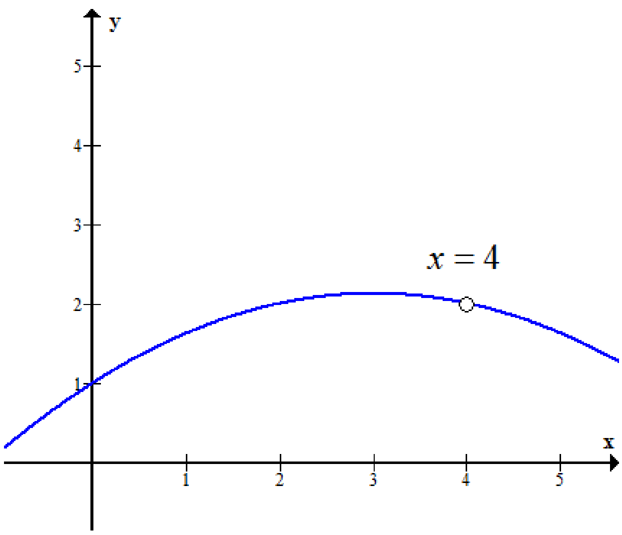
If any of these conditions fail, f is discontinuous.

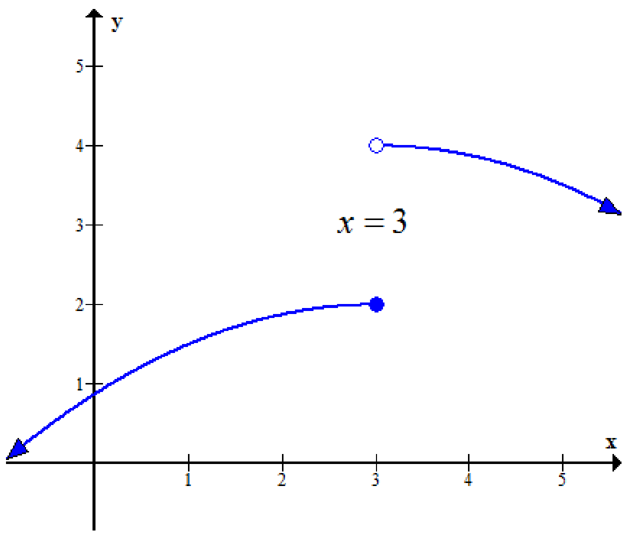
Note: From algebra you probably said a function is not continuous if when tracing the graph with your pencil, you have to lift your pencil off the paper. Even though this isn't a formal way of defining continuity, it's a good way of looking at it.

Continuity

Example 1.8

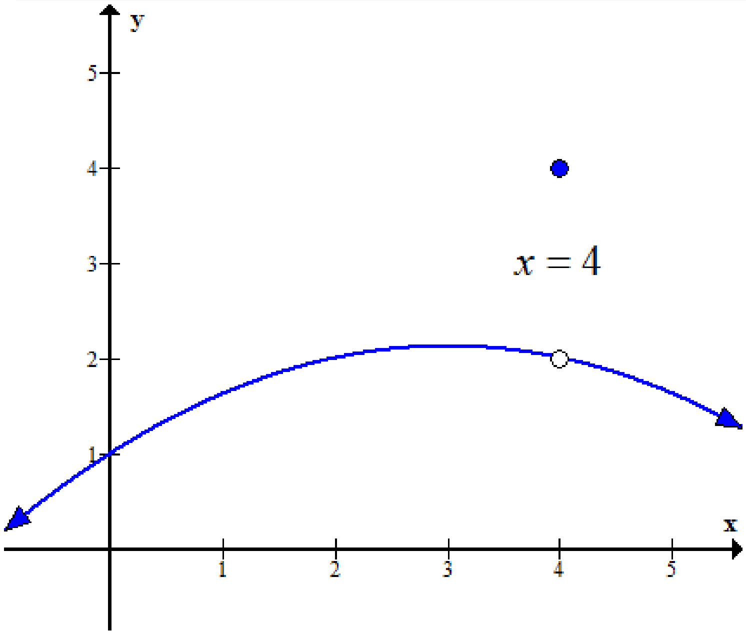
Examine the graphs and decide if they are continuous at the indicated point.





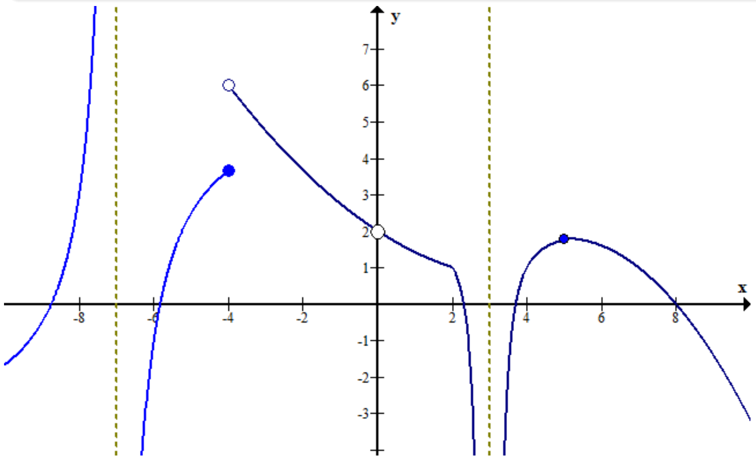
Example 1.9

Examine the graph and decide if they are continuous at the indicated point.



Example 1.10

Determine all the places where this graph is **NOT** continuous.



Example 1.11

Is $f(x) = \dfrac{x^2 - x - 2}{x - 2}$ continuous at $x = 2$?

Example 1.12

Is $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$ continuous at $x = 2$?

Example 1.13

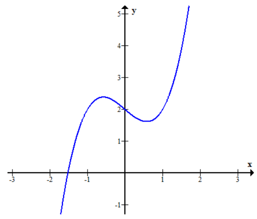
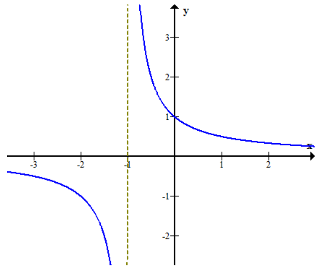
Is $f(x) = \begin{cases} 5 - 4x, & x \leq 1 \\ \sqrt{x} + 1, & x > 1 \end{cases}$ continuous at $x = 1$?

Example 1.14

Where is $g(x) = \begin{cases} x+1, & x < 1 \\ x^2 - 3x + 4, & 1 \leq x \leq 3 \\ \sqrt{7-x}, & x > 3 \end{cases}$ continuous?

Chapter - Limits 1.2 - Algebraic Limits and Continuity

Summary of Continuity

Type of Function	Where is it continuous?	Graphic Example
Polynomial	For all x	
Rational Function $y = \dfrac{P(x)}{Q(x)}$	For all x where $Q(x) \neq 0$	
Root Functions $y = \sqrt{ax+b}$	For all x where $ax + b \geq 0$