Business Calculus

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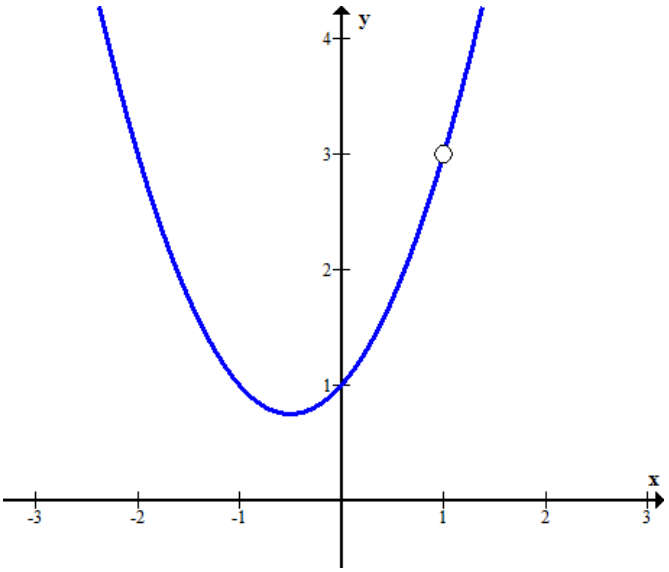
Look at the graph of $f(x) = \dfrac{x^3 - 1}{x - 1}$. Can you guess what the function values are approaching as $x \to 1$?



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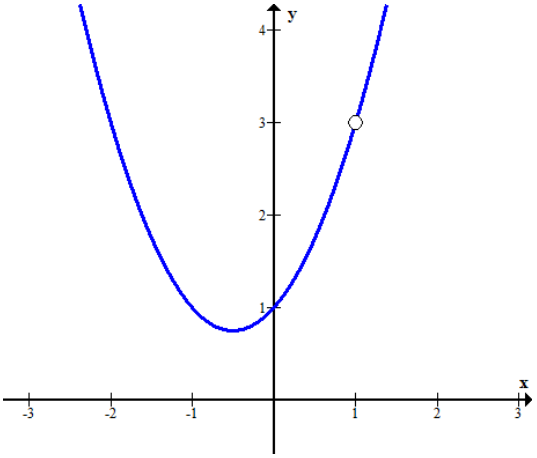


x	0.9	0.99	0.999		1.001	1.01	1.1
$f(x)$							

Limit

Definition 1.1

Let f be a function. As x takes values closer and closer (not equal) to a on both sides, the corresponding y values get closer and closer to L. Then the **limit** of $f(x)$ as x approaches a is L, written

Left and Right Hand Limits

Example 1.2

Consider $\lim_{x \to 2} x^2 - x + 2$

x	y	x	y
1	2	3	8
1.5	2.75	2.5	5.75
1.9	3.71	2.1	4.31
1.99	3.9701	2.01	4.0301
1.999	3.997001	2.001	4.003001
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

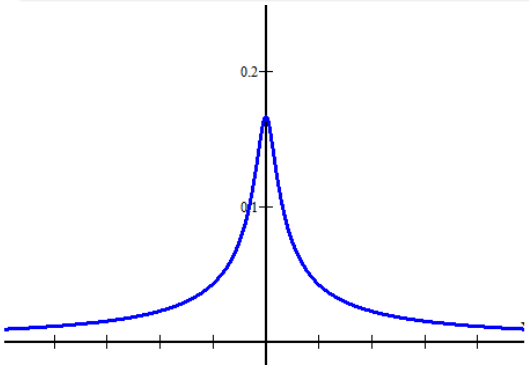
Theorem 1.3

As x approaches a, the limit of $f(x)$ is L if the limit from the left and right exist and both are L. In other words,

$$\textit{If } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L \textit{ then } \lim_{x \to a} f(x) = L$$

Example 1.4

The graph of $f(x) = \dfrac{\sqrt{x^2+9}-3}{x^2}$ is shown below. Find $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+9}-3}{x^2}$



x	y	x	y
.1	.1666	-.1	.1666
.01	.1667	-.01	.1667
.001	.166667	-.001	.16667
↓	↓	↓	↓
	.1666...		.1666...

Example 1.5

Let $g(x) = \begin{cases} x^2, & x < 2 \\ x+2, & x \geq 2 \end{cases}$

Find $\lim\limits_{x\to 2} g(x)$ by first drawing a graph.

Example 1.6

Let $g(x) = \begin{cases} x^2, & x \neq 2 \\ 2, & x = 2 \end{cases}$

Find $\lim_{x \to 2} g(x)$ by first drawing a graph.

Example 1.7

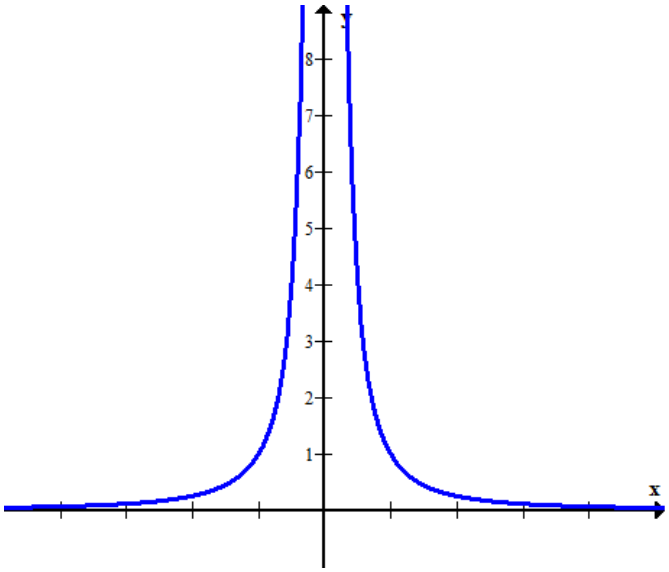
Graph $f(x) = \begin{cases} -x + 4, & x < 3 \\ x - 3, & x > 3 \end{cases}$. Then find $\lim_{x \to 3^-} f(x)$, $\lim_{x \to 3^+} f(x)$, and $\lim_{x \to 3} f(x)$,

Existence of Limits

The limit of f as x approaches a may not exist.

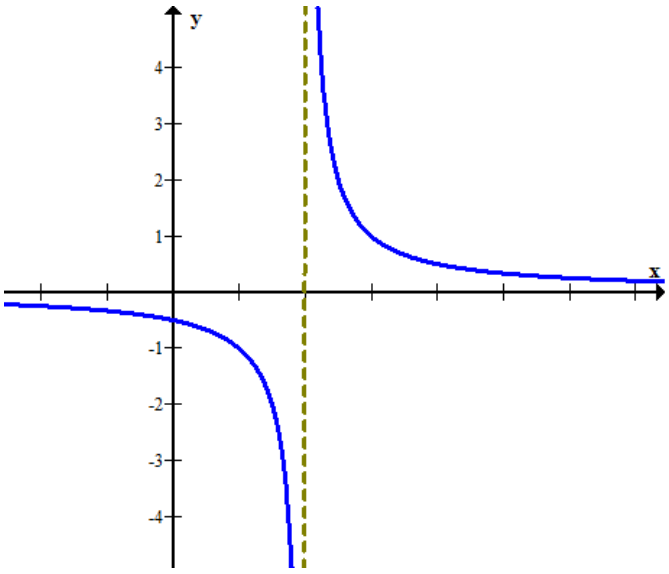
1. If $f(x)$ becomes infinitely large (positive or negative from both sides) as x approaches the number a from either side, then we write

$$\lim_{x\to a} f(x) = \infty \text{ or } \lim_{x\to a} f(x) = -\infty$$



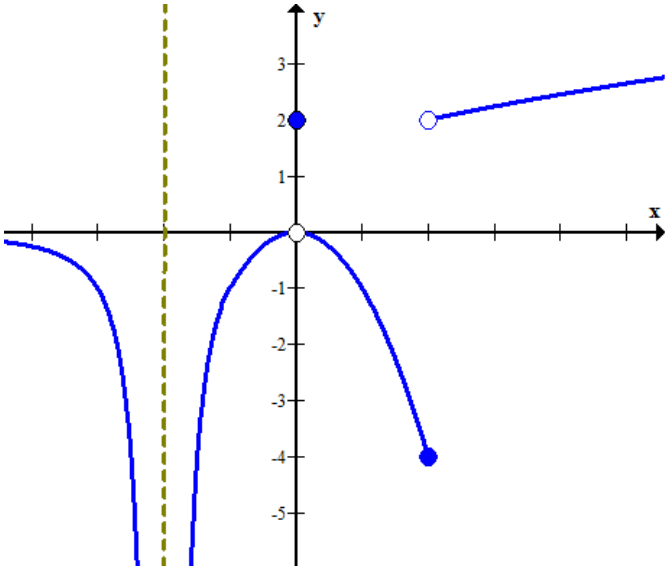
2. If $f(x)$ becomes infinitely large (positive) on one side of a and infinitely large (negative) on the other side of a, then

$$\lim_{x\to a} f(x) = \text{ Does not exist}$$

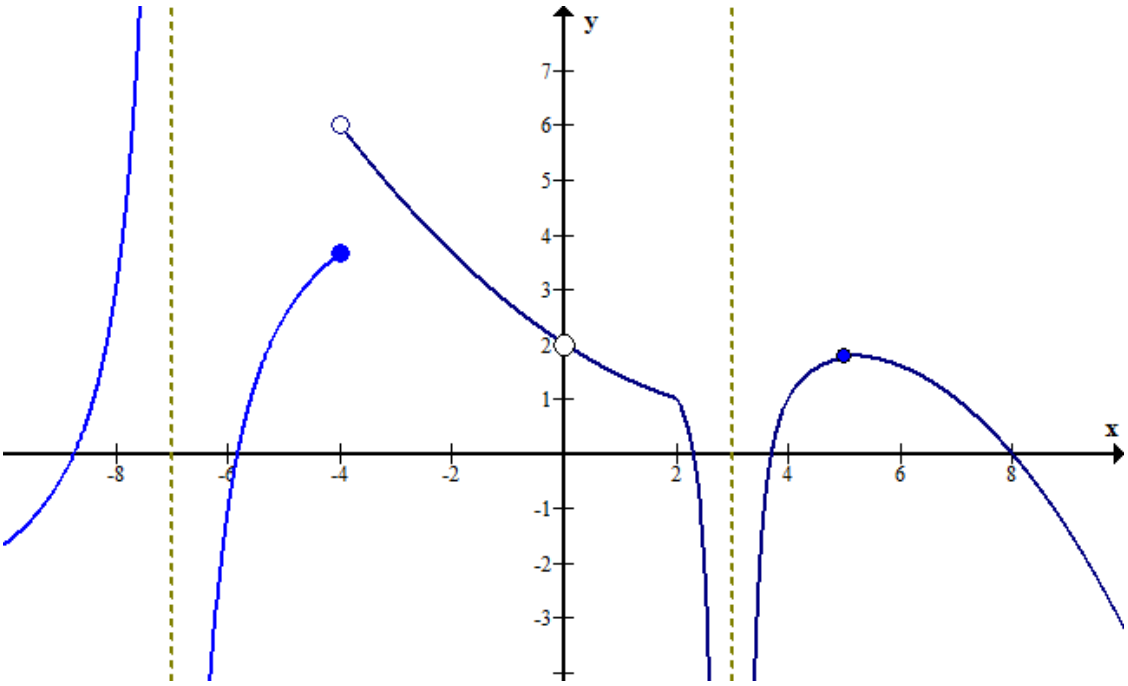


3 If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$ where $L \neq M$, then

$$\lim_{x \to a} f(x) = \text{ Does not exist}$$



Example



Example

Consider the function $f(x) = \dfrac{1}{x-3} + 4$. Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to 3} f(x)$ using a table and the graph.