This study guide is in no way exhaustive. As stated in class, any type of question from class, quizzes, exams, and homeworks are fair game. There's no information here about the word problems.

1. **Some Algebra Review**

   (a) **Factoring and Solving**

   i. **Quadratic Formula:** $ax^2 + bx + c = 0$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

   ii. **Difference of Two Squares**

   $$u^2 - a^2 = (u - a)(u + a)$$

   Careful! Sometimes you have to rewrite the equation, ex.

   $$-x^2 + 4 = 4 - x^2 = (2 - x)(2 + x)$$

   iii. **Factor Trinomials:** $ax^2 + bx + c$. Some examples

   $$x^2 + 10x + 16,\ 6x^2 + x - 12,\ 2x^2 - 6x - 10$$

   (b) **Logarithm Properties**

   i. $y = \log_b x$ is equivalent to $x = b^y$.

   ii. $\log_b b = 1$

   iii. $\log_b 1 = 0$, $\ln 1 = 0$

   iv. $\log_b(b^r) = r$, $\ln(e^r) = r$

   v. $\log_b(x^r) = r \log_b(x)$, $\ln(x^r) = r \ln(x)$

   vi. $\log_b(MN) = \log_b(M) + \log_b(N)$

   vii. $\log_b \left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

   viii. The domain of $\log_b(u)$ is $u > 0$

   ix. All the rules above hold for $\ln x$. Remember that $\ln x = \log_e x$

   x. Change of Base

   $$\log_a(M) = \frac{\ln M}{\ln a} = \frac{\log_b M}{\log_b a}$$

   This is useful if you're asked to find the derivative of $y = \log_5(x + 1)$

   (c) **Exponential Properties**

   i. $a^n a^m = a^{n+m}$

   ii. $(a^n)^m = a^{nm}$

   iii. $(ab)^n = a^n b^n$

   iv. $\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

   v. $a^{-n} = \frac{1}{a^n}$

   vi. $\frac{1}{a^{-n}} = a^n$

   vii. $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$

(d) **Properties of Radicals**

i. $\sqrt[n]{a} = a^{1/n}$

ii. $\sqrt[n]{a^m} = a^{m/n}$

2. Limits

(a) **Notation**

i. General Limit Notation: $\lim_{x\to a} f(x) = L$

ii. Left Hand Limit: $\lim_{x\to a^-} f(x) = L$

iii. Right Hand Limit: $\lim_{x\to a^+} f(x) = L$

iv. If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$ then $\lim_{x\to a} f(x) = L$

(b) **Limits at $\pm\infty$.** Assume all polynomials are in descending order.

i. $\lim_{x\to\infty} \frac{a}{x^r} = 0,\ r > 0$

ii. If $n > m$, then $\lim_{x\to\infty} \frac{ax^m + ...}{bx^n + ...} = 0$

iii. $\lim_{x\to\infty} \frac{ax^n + ...}{bx^n + ...} = \frac{a}{b}$

iv. If $m > n$, then $\lim_{x\to\infty} \frac{ax^m + ...}{bx^n + ...} = \pm\infty$

(c) **Evaluation Techniques**

i. If $f(x)$ is continuous, then $\lim_{x\to a} f(x) = f(a)$

$$\lim_{x\to 3} \sqrt{x^2+4} = \sqrt{3^2+4} = \sqrt{13}$$

ii. **Factor and Cancel**

If you evaluate a rational function and get $\frac{0}{0}$, then try to factor.

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3} \frac{(x+3)}{1} = 6$$

iii. **Rationalizing Numerators / Denominators**

Try this technique if you have radicals. Multiply top and bottom by the conjugate.

$$\lim_{x\to 4} \frac{\sqrt{x}-4}{x-16} = \lim_{x\to 4} \frac{\sqrt{x}-4}{x-16} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4} = \lim_{x\to 4} \frac{x-16}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+4}$$

iv. **Combine by Using Common Denominators**

Try this when you need to combine fractions within fractions

$$\lim_{x\to a} \frac{\frac{2}{x+4} - \frac{2}{a+4}}{x-a} = \lim_{x\to a} \frac{\frac{2}{x+4} - \frac{2}{a+4}}{x-a} \cdot \frac{(x+4)(a+4)}{(x+4)(a+4)} = \lim_{x\to a} \frac{2(a+4)-2(x+4)}{(x-a)(x+4)(a+4)}$$

$$= \lim_{x\to a} \frac{2a-2x}{(x-a)(x+4)(a+4)} = \lim_{x\to a} \frac{-2(x-a)}{(x-a)(x+4)(a+4)} = \lim_{x\to a} \frac{-2}{(x+4)(a+4)} = \frac{-2}{(a+4)(a+4)}$$

(d) **Piecewise Functions** - Know how to graph and evaluate Piecewise Functions. These are good ones to test your understanding of left-hand, right-hand, and general limits. They are also used to test your understanding of continuity.

(e) **Definition of Continuity**

A function f is continuous at $x = a$ if the following three conditions are satisfied:

i. $f(a)$ must exist

ii. $\lim_{x \to a} f(x)$ must exist

iii. $\lim_{x \to a} f(x) = f(a)$

(f) **Asymptotes**

i. **Vertical Asymptotes:**

The line $x = a$ is a vertical asymptote if any of the following occur:

$$\lim_{x \to a^-} f(x) = \pm\infty \text{ or } \lim_{x \to a^+} f(x) = \pm\infty$$

From algebra, you can find vertical asymptotes by

A. Reducing your rational function (no common factors in numerator and denominator).

B. Finding the x values that make the denominator 0.

ii. **Horizontal Asymptotes**

The line $y = b$ is a horizontal asymptote if either or both occur:

$$\lim_{x \to -\infty} f(x) = b \text{ or } \lim_{x \to \infty} f(x) = b$$

Refer to 2(b) of this guide for shortcuts on evaluating these limits.

iii. **Slant Asymptotes:**

Given the function $f(x) = \dfrac{P(x)}{Q(x)}$, a slant asymptote occurs when the degree of the numerator is 1 greater than the degree of the denominator.

$$\text{Ex. } f(x) = \frac{x^2 - 4}{x - 1}$$

You find the slant asymptote by doing long division.

3. **Derivatives**

   (a) **Limit Definition of a Derivative**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

   (b) **Tangent Lines**

   When asked to find the equation of a tangent line on f at $x = a$, you need two things: A point and a slope.

   i. You're usually given the x value. If they don't tell you the y value, you must plug the x value into $f(x)$ to get the y value. Now you have a point $(a, f(a))$

   ii. To find the slope m, you find $m = f'(a)$.

   iii. The equation of the tangent line is $y - f(a) = f'(a)(x - a)$. You may need to write this in slope-intercept form.

   (c) **Derivative Formulas**

   i. $\frac{d}{dx}(c) = 0$

   ii. $\frac{d}{dx}(f \pm g) = f' \pm g'$

   iii. $\frac{d}{dx}(x) = 1$

   iv. $\frac{d}{dx}(kx) = k$

   v. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

   vi. $\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1} \cdot f'(x)$

   vii. Product Rule: $(f\ g)' = f'g + fg'$

   viii. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

   ix. $\frac{d}{dx}(a^x) = a^x \ln a$

   x. $\frac{d}{dx}(e^x) = e^x$

   xi. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

   xii. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

   xiii. $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x) \cdot e^{f(x)}$

   xiv. $\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)} \cdot f'(x)$

   (d) **Critical Points**

   $x = c$ is a value of $f(x)$ if $f'(c) = 0$ or $f'(c)$ does not exist.

   (e) **Increasing / Decreasing**

   i. If $f'(x) > 0$ on an interval I, then $f(x)$ is increasing.

   ii. If $f'(x) < 0$ on an interval I, then $f(x)$ is decreasing.

   (f) **Concave Up / Concave Down**

   i. If $f''(x) > 0$ on an interval I, then $f(x)$ is concave up.

   ii. If $f''(x) < 0$ on an interval I, then $f(x)$ is concave down.

   (g) **Inflection Points**

   $x = c$ is an inflection point of f if

i. The point at $x = c$ must exist.

ii. Concavity changes at $x = c$

(h) **First Derivative Test to find Local (Relative) Extrema**

i. Find all critical value of $f(x)$ where $f'(c) = 0$

ii. Local Max at $x = c$ if $f'(x)$ changes from $(+)$ to $(-)$ at $x = c$.

iii. Local Min at $x = c$ if $f'(x)$ changes from $(-)$ to $(+)$ at $x = c$.

iv. If $f'(x)$ does not change signs, it's still an important point to plot. It may be a place where the slope is 0, a corner, an asymptote, a vertical tangent line, etc.

v. **Make sure you write your relative max and mins as points** $(c, f(c))$

(i) **Second Derivative Test to find Local (Relative) Extrema**

i. Find all critical value of $f(x)$ where $f'(c) = 0$

ii. Local Max at $x = c$ if $f''(c) < 0$

iii. Local Min at $x = c$ if $f''(x) > 0$

iv. **Make sure you write your relative max and mins as points** $(c, f(c))$

(j) **Absolute Extrema**

i. $(c, f(c))$ is an absolute maximum of $f(x)$ if $f(c) \geq f(x)$ for all x in the domain.

ii. $(c, f(c))$ is an absolute minimum of $f(x)$ if $f(c) \leq f(x)$ for all x in the domain.

(k) **Finding Absolute Extrema of a continuous** $f(x)$ **over** $[a, b]$

i. Find all critical values of $f(x)$ on $[a, b]$.

ii. Evaluate $f(x)$ at all critical values.

iii. Evaluate $f(x)$ at the endpoints, $f(a)$ and $f(b)$.

iv. The absolute max is the largest function value and the absolute min is the smallest function value.

(l) Differentials / Linearization

i. **Marginal Cost, Revenue, and Profit**

$$C(x + k) \approx C(x) + kC'(x)$$

$$R(x + k) \approx R(x) + kR'(x)$$

$$P(x + k) \approx P(x) + kP'(x)$$

ii. Differentials

A. Δx - is the true change in x

B. differential dx - is our independent variable that represents the change in x. We let $dx = \Delta x$.

C. Δy - is the true change in y

D. differential dy - is the estimated change in y

E. Formula:

$$\frac{dy}{dx} = f'(x) \text{ or } dy = f'(x) \cdot dx$$

4. **Summary of Curve Sketching** – Always start by noting the domain of $f(x)$

   (a) x and y intercepts

      i. x-intercepts occur when $f(x) = 0$

      ii. y-intercept occurs when $x = 0$

   (b) Find any vertical, horizontal asymptotes, or slant asymptotes.

      i. Vertical Asymptote: Find all x-values where $\lim_{x \to a} f(x) = \pm\infty$. Usually when the denominator is 0 and the numerator is not 0. Rational function MUST be reduced.

      ii. Horizontal Aymptotes: Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$. There are shortcuts based on the degree of the numerator and denominator.

      iii. Slant Asymptotes: Occurs when the degree of the numerator is one larger than the denominator. You must do long division to determine the asymptotes.

   (c) Find $f'(x)$

      i. Find the critical values, all x-values where $f'(x) = 0$ or when $f'(x)$ does not exist.

      ii. Plot the critical values on a number line.

      iii. Find increasing / decreasing intervals using number line

      iv. Use **The First Derivative Test** to find local maximums / minimums (if any exist). Remember to write them as points.

         A. Local Max at $x = c$ if $f'(x)$ changes from $(+)$ to $(-)$ at $x = c$.

         B. Local Min at $x = c$ if $f'(x)$ changes from $(-)$ to $(+)$ at $x = c$.

         C. Note: If $f'(x)$ does not change signs, it's still an important point. It may be a place where the slope is 0, a corner, an asymptote, a vertical tangent line, etc.

   (d) Find $f''(x)$

      i. Find all x-values where $f''(x) = 0$ or when $f''(x)$ does not exist.

      ii. Plot these x-values on a number line.

      iii. Find intervals of concavity using the number line

      iv. Find points of inflection

         A. Must be a place where concavity changes

         B. The point must exist (i.e, can't be an asymptote, discontinuity)

   (e) Sketch

      i. Draw every asymptote

      ii. Plot all intercepts

      iii. Plot all critical points (even if they are not relative extrema). They were critical points for a reason.

      iv. Plot all inflection points.

      v. Connect points on the graph by using information about increasing/decreasing and its concavity.

5. **Integrals**

(a) Definitions

i. **Antideriative:** An antiderivative of $f(x)$ is a function $F(x)$, where $F'(x) = f(x)$.

ii. **General Antiderivative:** The general antiderivative of $f(x)$ is $F(x) + C$, where $F'(x) = f(x)$. Also known as the **Indefinite Integral**

$$\int f(x)\,dx = F(x) + C$$

iii. **Definite Integral:**

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(b) Common Integrals

i. $\int k\,dx = kx + C$

ii. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

iii. $\int \frac{1}{x}\,dx = \ln|x| + C$

iv. $\int \frac{1}{kx+b}\,dx = \frac{1}{k}\ln|kx+b| + C$

v. $\int e^x\,dx = e^x + C$

vi. $\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$

vii. $\int e^{kx+b}\,dx = \frac{1}{k}e^{kx+b} + C$

viii. $\int a^x\,dx = \frac{a^x}{\ln a} + C$

ix. $\int a^{kx}\,dx = \frac{1}{k\ln a}a^{kx} + C$

x. $\int a^{kx+b}\,dx = \frac{1}{k\ln a}a^{kx+b} + C$

(c) Every integral must be written into the proper form in order to use the formulas. For example,

$$\int \sqrt{x}\,dx = \int x^{1/2}\,dx$$

$$\int \frac{3}{5x^4}\,dx = \int \frac{3}{5}x^{-4}\,dx$$

$$\int \frac{8}{\sqrt[5]{x^9}}\,dx = \int 8x^{-9/5}\,dx$$

(d) Finding Area under or between Curves