Exam II   Math 211, Business Calculus, Spring 2014   Name:

This is only a practice exam. It does not cover all the material that can be asked on the exam. Make sure you can do all the problems from the quizzes and lecture notes.

1. Find the derivative for each of the following functions. Do Not simplify

(a) $f(x) = \frac{1}{x^2} - \frac{2}{x} + 2x + 5 = x^{-2} - 2x^{-1} + 2x + 5$ $f'(x) = -2x^{-3} + 2x^{-2} + 2$

(b) $f(x) = 2\sqrt{x} + 5x^{10} + \frac{3}{x^2} = 2x^{1/2} + 5x^{10} + 3x^{-2}$ $f'(x) = x^{-1/2} + 50x^{9} - 6x^{-3}$

(c) $f(x) = 5^2$ $f'(x) = 0$

this is a constant

(d) $f(x) = \frac{2x^3 + 3x^2}{2x + 3}$ $f'(x) = \frac{(2x+3)(6x^2+6x) - (2x^3+3x^2)(2)}{(2x+3)^2}$

Division Bar

(e) $f(x) = (5x^2 + 2)(3x + x^3)$ $f'(x) = (5x^2+2)(3+3x^2) + (10x)(3x+x^3)$

(f) $f(x) = (2x^3 + x^2 + 2)^{20}$ $f'(x) = 20(2x^3+x^2+2)^{19}(6x^2+2x)$

(g) $f(x) = \underbrace{x^2}_{g}\underbrace{(x^4 + 1)^{10}}_{h}$ $f'(x) = \underbrace{x^2}_{g}\cdot\underbrace{10(x^4+1)^9\cdot 4x^3}_{h'} + \underbrace{(x^4+1)^{10}\cdot 2x}_{h\cdot g'}$

(h) $f(x) = \frac{(x^2 + 1)^3}{(x^2 - 1)^2}$ $f'(x) = \frac{(x^2-1)^2\cdot 3(x^2+1)^2\cdot 2x - (x^2+1)^3\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4}$

2. Given the function $h(x) = 3x^3 + 18^2 + 7$, find the critical numbers (just the x-value).

1) Find $h'(x)$: $h'(x) = 9x^2 + 36x$

2) Solve $h'(x) = 0$

$$9x^2 + 36x = 0$$
$$9x(x+4) = 0$$
$$\Rightarrow x = 0, -4$$

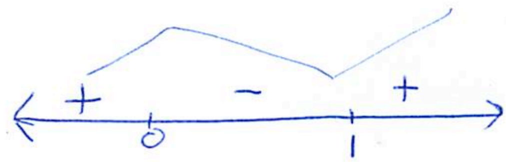
3. Given that $f(x) = 2x^3 - 3x^2$ has critical numbers $x = 0$ and $x = 1$, find the

(a) relative maximum and relative minimum (if they exist).

$f'(x) = 6x^2 - 6x \longrightarrow 6x^2 - 6x = 0$

$6x(x-1) = 0$

$x = 0, 1$



(b) intervals of increasing / decreasing

Increasing: $(-\infty, 0) \cup (1, \infty)$

Decreasing: $(0, 1)$

Max: $(0, 0)$

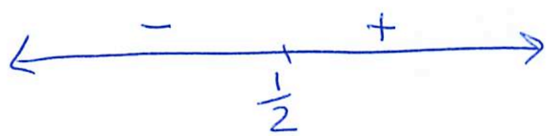
Min: $(1, -1)$

(c) Inflection Points

$f''(x) = 12x - 6 \longrightarrow 12x - 6 = 0$

$6(2x-1) = 0$

$x = 1/2$



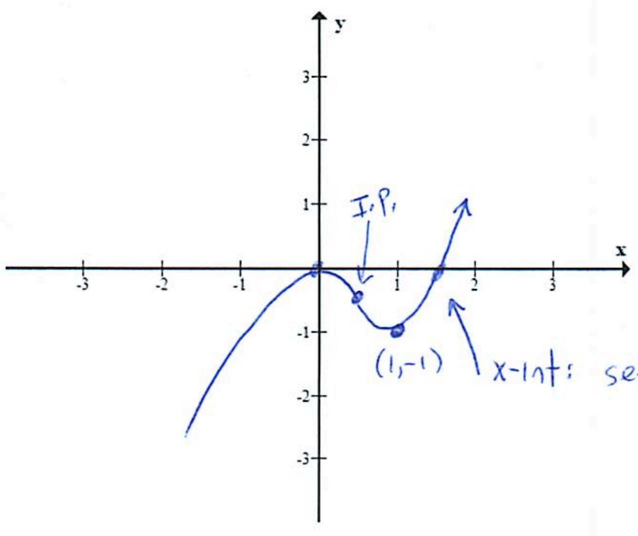
(d) Intervals of concavity

Concave Up: $(1/2, \infty)$

Concave Down: $(-\infty, 1/2)$

Inflection Point: $(1/2, -1/2)$

(e) Sketch the graph



x-int: set $2x^3 - 3x^2 = 0$

$x^2(2x-3) = 0$

$x = 0, \ x = 3/2$

4. Find the limit, if it exists

(a) $\lim_{x\to\infty} \dfrac{x^4+3x^2}{2x^5+x+7}$ $= 0$

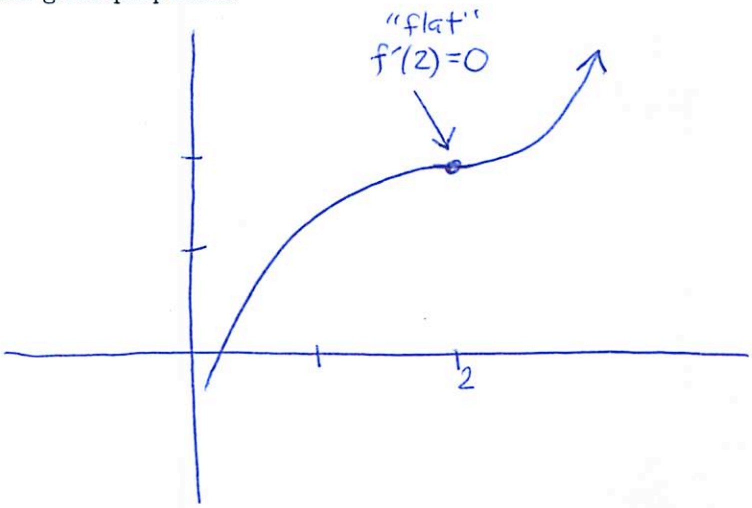
(b) $\lim_{x\to\infty} \dfrac{4x^3+4x}{2x^3-3x}$ $= \frac{4}{2} = 2$

(c) $\lim_{x\to\infty} \dfrac{x}{x+4}$ $= 1$

(d) $\lim_{x\to 2^+} \dfrac{1}{x-2}$ $= \infty$

5. Sketch the graph of a function having the given properties:

(a) $f(2) = 2$
(b) $f'(2) = 0$
(c) $f'(x) > 0$ on $(-\infty, 2)$
(d) $f'(x) > 0$ on $(2, \infty)$
(e) $f''(x) < 0$ on $(-\infty, 2)$
(f) $f''(x) > 0$ on $(2, \infty)$



6. Let $f(x) = \dfrac{x-1}{x-2}$, $f'(x) = \dfrac{-1}{(x-2)^2}$, and $f''(x) = \dfrac{2}{(x-2)^3}$

(a) Find the vertical asymptote

V.A. $X = 2$

It's important to write $X = ?$

same with $y = ?$

(b) Find the horizontal asymptote

H.A. $y = 1$

(c) Find the intervals where $f(x)$ is increasing or decreasing

$f'(x) = \dfrac{-1}{(X-2)^2}$ C.V. $X = 2$

$-$ $-$

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Decreasing: $(-\infty, 2) \cup (2, \infty)$

(d) Find the interval where $f(x)$ is concave up or down.

$f''(x) = \dfrac{2}{(X-2)^3}$ $f''(x)$ DNE when $X = 2$

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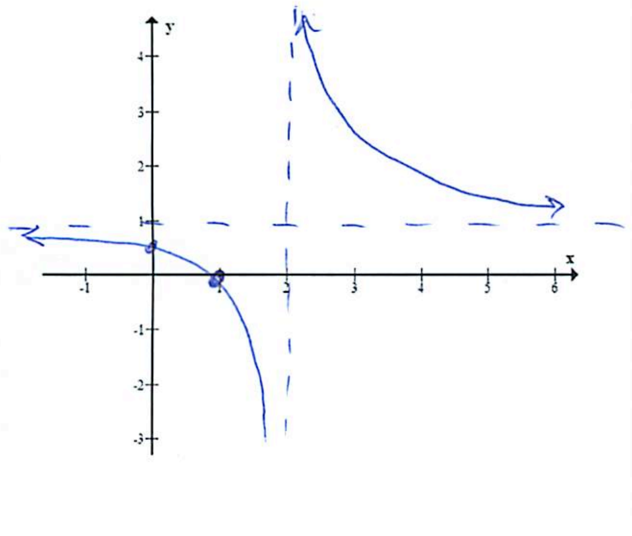
Concave Up: $(2, \infty)$
Concave Down: $(-\infty, 2)$

(e) Draw a rough sketch of the graph using ALL of the above information.



Helpful to find x,y intercepts

7. Suppose that $P(x) = -0.01x^2 + 60x - 500$ is the profit function from the manufacture and sale of telephones.

(a) What is the profit when you manufacture and sell 100 telephones?

$P(100) = -.01(100)^2 + 60(100) - 500 = \$5400$

(b) Is profit increasing or decreasing when 100 telephones have been sold? What is $P'(100)$.

Need $P'(x)$:

$P'(x) = -.02x + 60$

$P'(100) = -.02(100) + 60 = \$58/\text{phone}$

profit is increasing

(c) Assuming you've manufactured and sold 100 telephones, should you manufacture and sell the 101st telephone? Explain. Yes. Since profit is increasing at 100 phones you'd profit an additional \$58 to make the 101st phone.

(d) If you manufactured and sold 100 telephones, estimate the profit after manufacturing and selling the 101st telephone. Use only (a) and (b). What is the actual profit after selling 101 telephones?

$P(101) \approx P(100) + P'(100) = \$5400 + \$58 = \$5458$

Actual Profit: $P(101) = -.01(101)^2 + 60(101) - 500 = \$5457.99$

(e) Repeat (a) - (d) for when you manufacture and sell 4000 telephones.

$P(4000) = \$79{,}500$

$P'(4000) = -\$20/\text{phone}$

Do not sell the 4001st phone. You would lose an estimated \$20 in profit.

Estimate profit: $P(4001) \approx P(4000) + P'(4000) \approx \$79{,}480$

Actual Profit: $P(4001) = \cancel{\$79{,}480} \ \$79{,}479.99$