

ALL WORK should be organized to be readable and must be of sufficient depth to justify your answer. You must **SHOW CORRECT WORK** in order to get credit.

1. (15 points) Differentiate the following functions.

(a) $y = \ln 7 + x^2 \ln(e^{2x} + 1)$

(b) $y = \ln \left(\frac{e^x \sqrt{3x + 5}}{x^5 (2x^2 - 3)^4} \right)$

(c) $y = (3x^2 + 1)^x$ (You must use logarithmic differentiation!)

2. (10 points) A population of tribbles grows at a rate proportional to the number present. If you start out with 2 tribbles and the population triples every 5 hours, find the formula $Q(t)$ for the size of the population after t hours.

3. (10 points) Let $f(x) = \frac{\ln x}{x}$.

(a) Find and simplify $f''(x)$.

(b) Find the slope (only) of the line tangent to the graph of $f(x) = \frac{\ln x}{x}$ at its inflection point.

4. (15 points) A university is trying to determine what price to charge for football tickets. At a price of \$6 per ticket, it averages 70,000 people per game. For every increase of \$1, it loses 10,000 people from the average number. What price per ticket should be charged in order to maximize revenue?

5. (10 points) Determine the intervals on which $f(x) = xe^{-x^2}$ is increasing, and those on which it is decreasing.

Note: $\ln 2 \approx 7/10$.

6. (15 points) Let $f(x) = 4 - e^{-2x}$. Fill in the requested information and use the results to sketch a graph of f that includes that information.

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow -\infty} f(x) =$$

Horizontal asymptote(s):

$$f'(x) =$$

Interval(s) of increase:

Interval(s) of decrease:

$$f''(x) =$$

Interval(s) on which concave up:

Interval(s) on which concave down:

x -intercept(s):

y -intercept(s):

7. (15 points) Compute the following indefinite integrals.

$$\bullet \int \frac{10x^3 - 5x + 7\sqrt{x} - 5}{x^2} dx$$

$$\bullet \int \left(4 + \frac{e^{\sqrt{x}}}{\sqrt{x}} \right) dx$$

$$\bullet \int \frac{(\ln x)^3}{x} dx$$

8. (10 points) Solve the initial value problem $f'(x) = 3 - e^x$, $f(0) = 5$.